Chinese Journal of Polymer Science Vol. 25, No. 5, (2007), 501−507

Chinese Journal of Polymer Science

OBTAINING THE CRITICAL DRAW RATIO OF DRAW RESONANCE IN MELT SPINNING FOR POWER LAW POLYMER FLUIDS

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Abstract A direct difference method has been developed for Non-Newtonian power law fluids to solve the simultaneous non-linear partial differential equations of melt spinning, and to determine the critical draw ratio for draw resonance. The results show that for shear thin fluids, the logarithm of the critical draw ratio has a well defined linear relationship with the power index for isothermal and uniform tension melt spinning. When the power index approaches zero, the critical draw ratio points at unity, indicating no melt spinning can be processed stably for such fluids. For shear thick fluids, the critical draw ratio increases in a more rapid way with increasing the power index.

Keywords: Draw resonance; Melt spinning; Power law fluid; Numerical simulation.

INTRODUCTION

A number of mathematical methods have been applied to study draw resonance of melt spinning[1−17]. A previous paper of the author has shown that numerical simulation using different equations to solve the simultaneous partial non-linear differential equations of melting spinning is stable, therefore useful for obtaining the precise critical draw ratio of draw resonance for Newtonian fluids[18]. The stability problem of difference equations for melt spinning has been significantly simplified by computation with 19 digit long double precision. This success has enhanced the strength of direct difference methods that deserve more attention than other numerical methods, because they provide full information no other method is capable to provide. Not only for polymer processing, direct difference methods have also been found useful for polymer characterization using differential scanning calorimetry (DSC). DSC experiments can be described by a differential equation, complete DSC output curves can be obtained by a direct difference Runge-Kutta method[19, 20]. This paper however, extends the direct difference method refined in the previous paper[18] to obtain the critical draw ratio of draw resonance of melting spinning for Non-Newtonian power law fluids.

It shall be mentioned that Jung *et al.* have employed a finite difference method (FDM) to study the relationship between kinematic waves and draw resonance in viscoelastic isothermal spinning[11, 12], and to analyze the stabilizing effect of spinline cooling[13]. Lee *et al.* studied the effect of fluid viscoelasticity on the draw resonance dynamics[14] and the effect of the flow-induced crystallization on the transient behaviour of melt spinning of *i*PP[15]. These researches placed their focus on the practical applications of melt spinning, and only the situations either well above or well below the critical draw ratio of draw resonance were addressed. For these studies, the stability and accuracy are not an issue. However, the stability and accuracy of a different scheme become absolutely crucial when one aims at obtaining the critical draw ratio of draw resonance of a melt spinning system.

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Received August 22, 2006; Revised November 9, 2006; Accepted December 15, 2006

THE GOVERNING EQUATIONS FOR MELT SPINNING OF POWER LAW FLUIDS

The governing equations of melt spinning consist of a constitution equation and a continuity equation,

$$\frac{\partial v}{\partial x} = \frac{F}{A\beta} \tag{1}$$

$$\frac{\partial A}{\partial t} + \frac{\partial (Av)}{\partial x} = 0 \tag{2}$$

where, A and v denote the cross-sectional area and velocity of a filament section at time t and distance x from the spinneret. F represents the spinning tension and is a constant along the spinline but varies with time by definition of a uniform tension spinning. The symbol β is for elongational viscosity; for a power law fluid it reads:

$$\beta = \beta_0 \left(\frac{\partial v}{\partial x}\right)^{p-1} \tag{3}$$

Combining Eq. (1) and Eq. (3) leads to:

$$\left(\frac{\partial v}{\partial x}\right)^{p} = \frac{F}{A\beta_0} \tag{4}$$

The governing equations can be expressed in terms of non dimensional variables:

$$\frac{\partial \psi}{\partial \zeta} = \frac{\xi}{\lambda^{1/p}} \tag{5}$$

$$\frac{\partial \lambda}{\partial \tau} + \frac{\partial (\lambda \psi)}{\partial \zeta} = 0 \tag{6}$$

where, the non-dimensional parameters are defined as follows:

Distance from spinneret	$\zeta = x/L$
Time	$\tau = tV_0/L$
Cross sectional area	$\lambda(\zeta, \tau) = A/A_0$
Velocity of a filament	$\Psi(\zeta, \tau) = v/V_0$

Spinning tension that varies with time but a constant along the spinline by definition of a uniform tension spinning $\xi(\zeta, \tau) = \xi(\tau) = [F/(A_0\beta_0)]^{1/p} L/V_0$

where, L, A_0 and V_0 are the length of a spinline, *i.e.* the distance from the spinneret to the take-up bobbin, the cross-sectional area at the spinneret and the extrusion rate, respectively.

The governing equations have steady state solutions[14]:

For Newtonian fluids where $p = 1$,

$$\psi = \exp(\xi\zeta) \tag{7}$$

$$\lambda = \frac{1}{\exp(\xi\zeta)} \tag{8}$$

$$\xi = \ln \psi_{\mathrm{w}} \tag{9}$$

For power law fluids where $p \neq 1$,

$$\psi = \left[\left(\psi_{\mathrm{w}}^{1-q} - 1\right)\zeta + 1\right]^{\frac{1}{1-q}} \tag{10}$$

$$\lambda = \left[\left(\psi_{\mathrm{w}}^{1-q} - 1\right)\zeta + 1\right]^{\frac{-1}{1-q}} \tag{11}$$

$$\xi = \frac{\psi_{\mathrm{w}}^{1-q} - 1}{1-q} \tag{12}$$

where $q = 1/p$, and ψ_{w} is the draw ratio.

BOUNDARY AND INITIAL CONDITIONS

The same non dimensional boundary condition and initial condition as those used for the previous work are adopted in this study[16].

FINITE DIFFERENCE METHODS AND COMPUTIONAL PROCEDURES

The difference scheme employed in this study reads:

$$\psi_{i+1,j} = \psi_{i,j} + \left(\frac{\xi_j}{\lambda_{i,j}}\right)^q \Delta\zeta \tag{13}$$

$$\lambda_{i+1,j} = \frac{\omega\lambda_{i,j}\psi_{i,j} - (\lambda_{i,j} - \lambda_{i,j-1})}{\omega\psi_{i+1,j}} \tag{14}$$

where, $\omega = \Delta\tau/\Delta\zeta$ represents mesh ratio of the difference scheme. This scheme has been proven most stable for Newtonian fluids. The computational procedures can be referenced from the previous work of the author[16].

RESULTS AND DISCUSSION

Displayed in Figs. 1 and 2 are the responses, expressed in terms of variation percentage, of the cross sectional area of the taken up filament to a 0.01% step increment of the take up velocity for a power law fluid with its power index being 0.5. For Fig. 1, the initial draw ratio prior to the 0.01% step increment is 4.7, and the fluctuation of cross sectional area is observed to decay with time, leading to eventual elimination because the draw ratio is lower than the critical draw ratio for draw resonance. On the other hand, the fluctuation,

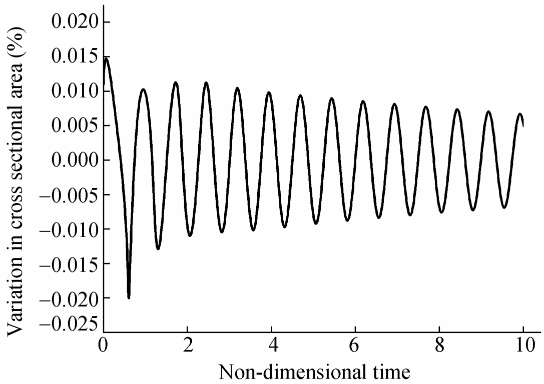

Fig. 1 Response of the cross-sectional area of taken-up filament to a 0.01% small step increment in the take-up velocity decays with time for fluids with power index 0.5
The draw ratio prior to the step increment is 4.7, lower than the critical draw ratio ($\Delta\tau = \Delta\zeta = 0.0001$).

as shown in Fig. 2, expands with time leading to an unstable spinning due to the draw ratio 5.1 over the critical draw ratio. An initial rather irregular downward sharp peak is observed for both Figs. 1 and 2, and the variation becomes regular sinusoidal like peaks at higher non-dimension time, τ.

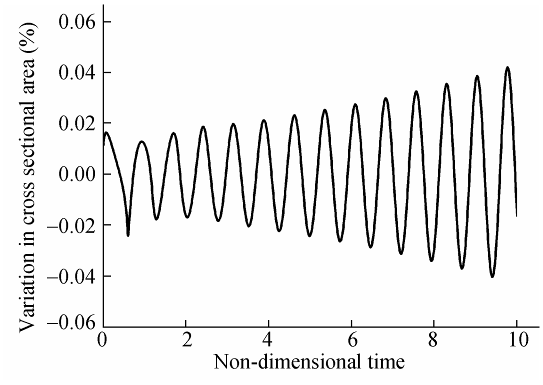

Fig. 2 Response of the cross-sectional area of taken-up filament to a 0.01% small step increment in the take-up velocity expands with time for fluids with power index 0.5
The draw ratio prior to the step increment is 5.1, greater than the critical draw ratio ($\Delta\tau = \Delta\zeta = 0.0001$).

Figures 3 and 4 display the responses for a power law fluid with its power index being 1.5 for two draw ratios 140 and 150 respectively. It is interesting to note that there is an initial period for the fluctuation to expand for the first few peaks, then to decay with time monotonically for Fig. 3. This is compared with the draw resonance for which an ever expanding fluctuation is observed in Fig. 4. By observing whether it decays or expands with time for a series of computations for different draw ratios, the critical draw ratio is searched and determined. These results have shown that the deference method is a reliable method available for obtaining the critical draw ratio for draw resonance for power law fluids.

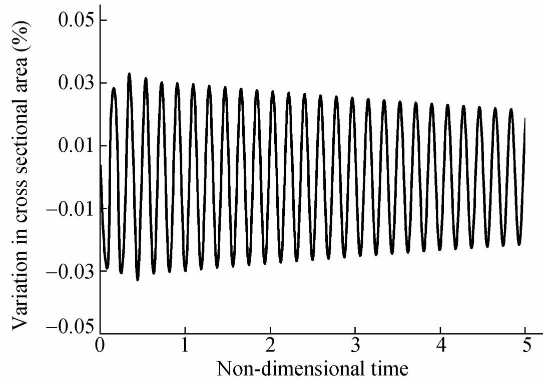

Fig. 3 Response of the cross-sectional area of taken-up filament to a 0.01% small step increment in the take-up velocity decays with time for fluids with power index 1.5
The draw ratio prior to the step increment is 140, lower than the critical draw ratio ($\Delta\tau = \Delta\zeta = 0.0001$).

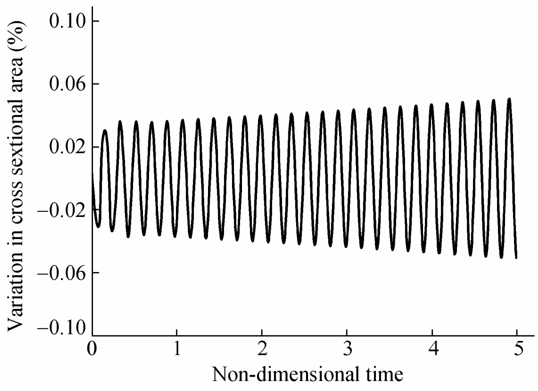

Fig. 4 Response of the cross-sectional area of taken-up filament to a 0.01% small step increment in the take-up velocity expands with time for fluids with power index 1.5
The draw ratio prior to the step increment is 150, greater than the critical draw ratio ($\Delta\tau = \Delta\zeta = 0.0001$).

It shall be mentioned that for computations for fluids with low p, the first two tentative ξ values should be carefully selected otherwise errors could occur to disable computation. A convenient practice is to choose two tentative values close to the ξ value for the next higher p. This is to say that one can adopt the two tentative ξ values for spinning of fluids with $p = 0.3$ from the computed ξ value for spinning of fluids with $p = 0.4$; to determine the two tentative values for the case of $p = 0.2$ from the case of $p = 0.3$, and so on and so forth.

The so determined critical draw ratio values for a range of power law fluids using 4 difference mesh schemes are listed in Table 1. According to whether $p < 1$ or $p > 1$, power law fluids are classified as shear thin and shear thick fluids with $p = 1$ for Newtonian fluids, respectively. For shear thin fluids, it is seen that the critical draw ratio approaches to a fixed value quickly with increasing the fineness of the mesh. The critical draw ratio has a well defined linear relationship with the power index p, with its origin pointing at draw ratio 1. This suggests that for infinitely shear thin fluids, no melt spinning could proceed as the spinning system is intrinsically instable.

Table 1. Critical draw ratio for draw resonance as obtained by the direct difference method and that as described by Eq. (15)

Power p	ψ_w^{cr} ($\Delta\tau = \Delta\zeta = 0.001$)	ψ_w^{cr} ($\Delta\tau = \Delta\zeta = 0.0004$)	ψ_w^{cr} ($\Delta\tau = \Delta\zeta = 0.0001$)	ψ_w^{cr} ($\Delta\tau = \Delta\zeta = 0.00004$)	$\psi_w^{cr} = \exp(3.065\,p)$
0.01	1.1	1.1	1.06		1.031
0.02	1.1	1.1	1.11		1.062
0.04	1.2	1.2	1.21		1.128
0.07	1.4	1.3	1.34		1.272
0.1	1.5	1.5	1.51		1.351
0.2	2.0	2.0	2.0		1.825
0.3	2.7	2.7	2.7		2.464
0.4	3.6	3.6	3.6		3.329
0.5	4.8	4.8	4.8		4.496
0.6	6.4	6.3	6.3		6.074
0.7	8.4	8.4	8.3		8.204
0.8	11.2	11.1	11.1		11.081
0.9	15.1	15.0	14.9		14.968
1.0*	–	–	–	20.218*	20.218
1.1	28	28	28	28	27.309
1.2	41	40	39	39	36.888
1.3	61	59	58	58	49.826
1.4	96	91	89	89	67.303
1.5	167	151	145	145	90.909
1.6	339	280	259	256	122.795
1.7	1140	645	539	503	165.864
1.8	> 100000	> 100000	1537	1382	224.041
1.9	–	–	> 100000	8592	302.622
2.0	–	–	–	> 100000	408.765

* The difference equations used in this study can not be applied if $p = 1$; The previous paper has shown the critical draw ratio 20.218 for Newtonian fluids[16].

However, for shear thick fluids, there are two phenomena observed. The first one is that much finer mesh is required to obtain the critical draw ratio satisfactory to the desired level of accuracy with increasing the power index p. For $p < 1.5$, it is observed that the critical draw ratio approaches a stable value with increasing the mesh fineness; but for $p >= 1.6$, the saturation get slower, indicating that finer mesh is required to improve the accuracy of the critical draw ratio. This is probably because that higher draw ratio is involved in these computations, the changes in the cross sectional area and velocity for a given $\Delta\tau$ (or $\Delta\zeta$) get steeper with increasing draw ratio, thus finer mesh is required to maintain the same level of accuracy. The second phenomenon is the so determined critical draw ratio increases with increasing the power index in a more rapid way than that observed for shear thin fluids, reaching over 1000 as the power index reaches 1.8. This fact suggests that there is no need to discuss the stability of spinning for fluids with its power index greater than 1.8 from practical point of view. Such a spinning system is mostly stable, it is not possible for draw resonance to occur within the range of practical draw ratio.

The logarithm of the critical draw ratio against the power index is plotted in Fig. 5. Clearly the plot is linear for shear thin fluids but bends upwards rapidly with increasing the power index for the region of p greater than unity.

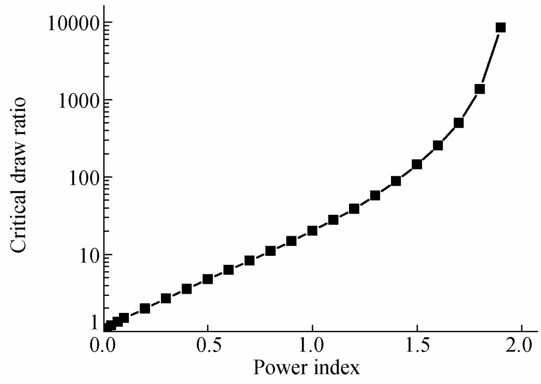

Fig. 5 Plot of the critical draw ratio determined by the numerical simulation versus the power index, p

There is a need to compare the results obtained by this direct difference method with previous studies. Pearshon and Shah[5, 6] used an eigenvalue method, and Ishihara and Kase[7, 8] also employed a direct difference method to obtain the critical draw ratio. They both failed to conclude that the critical draw ratio approaches unity for infinitely shear thin fluids ($p \rightarrow 0$).

A previous paper[14] of the author developed a draw mode theory to interpret the occurrence of draw resonance. Based on this theory, the stability of a spinline can be analyzed from its spinline profile at steady state. This leads to a relationship for the critical draw ratio and power index as:

$$\psi_{\mathrm{w}}^{\mathrm{cr}} = \mathrm{e}^{3.065p} \tag{15}$$

By comparing Eq. (15) and Table 1 and Fig. 5, one concludes that the equation agrees with the critical draw ratio obtained by the direct difference method very well for shear thin fluids, but not for shear thick fluids. Perhaps, this is because there is a factor suppressing draw resonance in melting spinning of shear thick fluids; draw resonance is basically a characteristic of shear thin fluids.

The unique advantage for the direct difference method is its capability to obtain full information of the process. One example is shown in Fig. 6, where the period of fluctuation after turbulence is plotted against the power index. Clearly, the period decreases with increasing the power index. The period is around 1.0 for fluids with the power index 0.01 and becomes around 0.1 for fluids with the power index 1.7.

Computation for the mesh scheme $\Delta\tau = \Delta\xi = 0.0001$ took around 4 h for a full run with final τ being 100 by a personal computer with 1.8 GHz CPU frequency. This is far longer than that for Newtonian fluids, presumably due to the computation involving power functions.

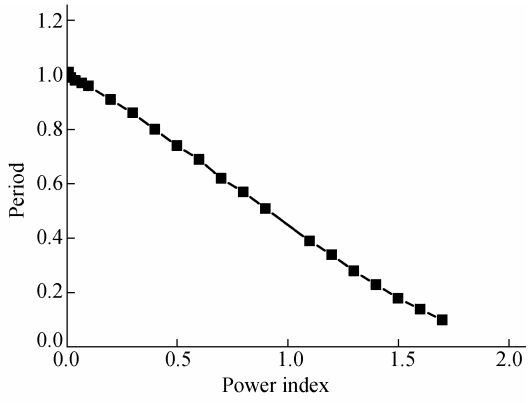

Fig. 6 Plot of the period of variation for the cross sectional area against the power index, p

CONCLUSIONS

The direct difference scheme adopted to Newtonian fluids has been applied to Non-Newtonian power law fluids to obtain the critical draw ratio of draw resonance of melt spinning. The results show the difference scheme is stable. The critical draw ratio has been determined for fluids with a range of power index from $p = 0.01$ to 2.0. It is found that for shear thin fluids, the logarithm of the critical draw ratio has a well defined linear relationship with the power index for isothermal and uniform tension melt spinning. When the power index approaches zero, the critical draw ratio points at unity, indicating no stable melt spinning can be achieved for such fluids. For shear thick fluids, the critical draw ratio increases with increasing the power index in a more rapid way. From the advantage that a direct difference method is capable to provide full information of a process, it is expected that direct difference methods will receive more and more attention.

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